Mathematics 1210 PRACTICE EXAM I Fall 2005

Answers for Extra Problems

1. $F(x) = \int (x+1)(3x-2)\mathrm{d}x = x^3 + \frac{1}{2}x^2 - 2x + C$ and $C = F(0) = 5$ implies $F(x) = x^3 + \frac{1}{2}x^2 - 2x + 5$.

2. $f(x) = \int (x^4 - 2x^3 + 1)dx = \frac{1}{5}x^5 - \frac{1}{2}x^4 + x + C$ where $f(1) = \frac{1}{5} - \frac{1}{2} + 1 + C = 0$ implies $C = -\frac{7}{10}$. Therefore, $f(x) = \frac{1}{5}x^5 - \frac{1}{2}x^4 + x - \frac{7}{10}$.

3.

$$x'(t) = \frac{dx}{dt} = \int (2t^2 - 3t + 1)dt = \frac{2}{3}t^3 - \frac{3}{2}t^2 + t + C$$

and $x'(0) = C = 6$, so $x'(t) = \frac{2}{3}t^3 - \frac{3}{2}t^2 + t + 6$. Then,

$$x(t) = \int \left(\frac{2}{3}t^3 - \frac{3}{2}t^2 + t + 6\right) dt = \frac{1}{6}t^4 - \frac{1}{2}t^3 + \frac{1}{2}t^2 + 6t + C$$

and $x(0) = C = 0$ imply $x(t) = \frac{1}{6}t^4 - \frac{1}{2}t^3 + \frac{1}{2}t^2 + 6t$.

4. The velocity function $v(t) = \int a(t)dt = \int t dt = \frac{1}{2}t^2 + C$ and $v(0) = 40$, so $v(t) = \frac{1}{2}t^2 + 40$. The position function $x(t) = \int v(t)dt = \int (\frac{1}{2}t^2 + 40)dt = \frac{1}{6}t^3 + 40t + C$, but $x(0) = C = 0$, so $x(t) = \frac{1}{6}t^3 + 40t$. Finally, 3 seconds is $\frac{3}{60^2}$ hours, so your position after 3 seconds is

$$x\left(\frac{3}{60^2}\right) = \frac{1}{6}\left(\frac{3}{60^2}\right)^3 + 40\left(\frac{3}{60^2}\right) > 40\left(\frac{3}{60^2}\right) = \frac{1}{30} > \frac{1}{100} = .01,$$

so yes, you do make it through the light! (Did you need to accelerate beyond 40 m/h?)

5. $v(y) = \int \frac{v_0}{D} dy = \frac{v_0}{D}y + C$ and $v(D) = v_0 + C = v_0$, so $C = 0$. Therefore, $v(y) = \frac{v_0}{D}y$.